[13.32] Show that every finite group **G** has a faithful representation in GL(*n*) where *n* is the order of **G.**

**Solution**

**Part A**. Show T is a representation

This proof of this part is just an elaboration of Robin’s method, which is very slick.

Let **G** = {*g*1, …, *g*n}. A representation is a group homomorphism, a function that preserves the group structure:

For all 

Thus, T(*gi*) is an *n* x *n* matrix. I use Penrose’s hint to label the rows and columns of matrix *T*(*gi* ) to indicate that the matrix takes *gs* to *gr* :

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Matrix T(*gj* ) can be written

,

matrix *T*(*gi* *gi* ) can be written

 ,

and the product matrix *T*(*gi* )*T*(*gj* ) is

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A strategy to define *T* such that *T*(*gi* *gj* )= *T*(*gi* )*T*(*gj* ) is to put as many zeros as possible into the matrix so that the calculation becomes simpler. To that end, define

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This matrix has precisely one 1 in every row and every column. The element  of the matrix *T*(*gi* )*T*(*gj* ) then becomes

.

That is,  ✔

**Part B**  Show *T* is faithful

*T* is faithful if it is one-to-one; i.e., if  So, suppose  



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